A METHOD OF DERIVING YIELD–NUTRIENT RESPONSE SURFACES FROM SIMPLE FERTILIZER EXPERIMENTS

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(met Opsomming in Afrikaans)

Summary

Quadratic, $Y = a+bX_1+cX_2+dX_1{}^2+eX_2{}^2+fX_1X^2$, and rectangular hyperbolic, $Y = (a' + b'/X_1+c'/X_2)^{-1}$, regression surfaces may both satisfactorily describe the response of crop yield to increasing amounts of two nutrients, X_1 and X_2, provided that nutrient levels are restricted to the range where 'diminishing returns' are exhibited. Use of the relationship $Y^{-1} = a' + b' X_1{}^{-1} + c' X_2{}^{-1}$, which yields a plane surface, enables the hyperbolic regression to be derived from only four treatments, a 'low' and a 'high' level of X_1 combined factorially with a 'low' and a 'high' level of X_2. Both the quadratic and the plane form of the hyperbolic model were fitted by least squares to results from three nitrogen, phosphorus experiments on maize. Goodness of fit was in each case somewhat better with the quadratic than with the hyperbolic regression. However, the simplicity of the sort of trial required to derive hyperbolic response surfaces should enable information for use in estimating economically optimal quantities of fertilizer to be obtained from comparatively large numbers of sites and seasons.

Introduction

Fertilizer often constitutes a substantial part of the cost of growing a crop; in southern Africa fertilizer accounts for a third to a half of the direct costs of production of maize (*Zea mays* L.). So the economically optimal amounts of fertilizer for each farmer's circumstances should be determined as precisely as possible. Such determinations require response curves of crop yield to increasing amounts of a single nutrient, or appropriate response surfaces where two nutrients are normally applied in substantial quantities, as for example are nitrogen and phosphorus to maize in southern Africa (Farina, Mapham and Channon, 1975).

The relationship between crop yield and increasing amounts of a fertilizer nutrient is commonly assumed to be smoothly curvilinear (e g Heady and Pesek, 1957; Anderson, 1957; Jones and Hocknell, 1962), but there is evidence of yield increasing linearly at first and then remaining more or less constant with increase in the amount of a nutrient, with a fairly abrupt transition from the rising to the horizontal portion of the response curve (Boyd, 1972). Nevertheless, it would seem that experimental data can usually be adequately described by curvilinear models. Thus, quadratic surfaces, $Y = a+bX_1+cX_2+dX_1{}^2+eX_2{}^2+fX_1X_2$ (Y, crop yield; X_1, level of one nutrient; X_2, level of a second nutrient), have given a satisfactory fit to a variety of experimental results (Goldsworthy, 1967a, b; Garaudeaux, 1973; Farina *et al*, 1975). The effects of increasing amounts of applied nutrients may be markedly influenced by factors such as season, soil conditions, cultivar and husbandry of the crop. On the other hand, fairly elaborate field experiments, requiring sophisticated management, are needed to derive response surfaces of the quadratic type, so they are probably only practicable at a few sites and on a limited number of occasions.

This paper suggests a method of using comparatively simple field trials to derive response surfaces of similar shape to the generally most useful portion of the quadratic response surface. The method might therefore be useful for obtaining data from a range of farm conditions and seasons. However, its use is limited to situations where the relationship between yield and amounts of fertilizer nutrients can be satisfactorily approximated by a curvilinear model, and in any case it should probably be regarded mainly as a means of supplementing the information obtained from more elaborate experiments conducted on experimental stations.

Theory

The economically optimal quantity of an applied nutrient can be expected to lie within a range where the yield *vs* nutrient response curve exhibits 'diminishing returns', i e the part of the classical production where, symbolically, $dY/dx > 0$, but $d^2Y/dx^2 < 0$ (cf Jones and Hocknell, 1962). Thus the curve might often be satisfactorily approximated by the rectangular hyperbolic relationship, $Y = (a+b/X)^{-1}$. As an example of the use of such a relationship Major and Daynard (1972) have shown its close fit to changes in leaf area per unit land area with increasing plant density in a number of maize crops. The relationship has the useful feature that the reciprocal of Y is linearly related to the reciprocal of X, so that the parameters of the relationship can be derived from two levels of X. By analogy, the relationship, $Y = (a+b/X_1+c/X_2)^{-1}$, could yield a regression surface of a satisfactory general shape for the response of yield to the joint effect of increasing levels of two nutrients. Here the use of the reciprocals of Y and X leads to a plane surface of the form $Y^{-1} = a+bX_1{}^{-1}+cX_2{}^{-1}$, so that a hyperbolic response surface could be derived from only four treatments, a 'low' and a 'high' level of X_1, combined factorially with a 'low' and a 'high' level of X_2. Clearly, the levels of both of the nutrients must lie within the diminishing returns range.

Because of its smaller number of terms, $Y = (a+b/X_1 + c/X_2)^{-1}$ is a less flexible function than $Y = a+bX_1 + cX_2 + dX_1^2 + eX_2^2 + fX_1X_2$, so seems likely to give a statistically poorer fit to experimental data.

Examples

Testing the method requires experiments with at least three levels of each of the two nutrients, excluding any nil levels, because the reciprocal of nought is meaningless in this context. Data from three maize fertilizer experiments reported by Farina *et al.* (1975), kindly supplied by the authors, have been used.

The experiments were carried out at different sites in Natal, South Africa. Two were unreplicated 4^3 nitrogen, phosphorus, potassium factorials, and one was a 5^2 nitrogen, phosphorus factorial with two replicates. The application of potassium had little effect on yield so the two experiments which included potassium could be regarded as nitrogen, phosphorus trials, each with four replicates. With the omission of nil levels of nitrogen or phosphorus these two experiments became 3^2 nitrogen, phosphorus factorials with four replicates, giving a total of 36 plots per experiment, and the third experiment a 5 x 4 nitrogen, phosphorus factorial with two replicates, making 40 plots in all (Table 1).

TABLE 1 Levels of nutrients and number of replicates selected for the present study from three factorial fertilizer experiments reported by Farina et al *(1975)*

Site and type of soil	Nutrient levels, N	kg ha^{-1} P	Number of replicates
Normandien	60	30	
clay loam	120	60	4
	180	120	
	60	25	
Winterton	120	50	4
clay loam	180	75	
	50	20	
Dundee	100	40	
sandy loam	150	80	2
	200	160	
	250	–	

TABLE 2 Levels of nutrients which, combined factorially, gave the four treatments used to derive a hyperbolic response surface for each site

Site	Nutrient levels, N	kg ha^{-1} P
Normandien	60	30
	180	120
Winterton	60	25
	180	75
Dundee	50	20
	250	160

Quadratic regressions, $Y = a+bN+cP+dN^2+cP^2+fNP$ (N, nitrogen; P, phosphorus), of grain yield per plot on nutrient levels were fitted by least squares to 36 variates from the experiments at Normandien and Winterton and to 40 variates from the experiment at Dundee. The plane form of the hyperbolic relationship, $Y^{-1} = a' + b'N^{-1} + c'P^{-1}$, was fitted by least squares to the values from four treatments in every experiment (Table 2), which with four replicates gave 16 variates for Normandien and Winterton respectively, and with two replicates gave eight variates for Dundee. Hyperbolic response surfaces, $Y = (a' + b'/N + c'/P)^{-1}$, were constructed, using values of a', b' and c' obtained from the fitting procedure. Grain yields were predicted by means of both the quadratic and the hyperbolic equations (Table 3).

Goodness of fit, shown by the R^2 values (Table 3), was always somewhat greater with the quadratic than with the hyperbolic response surface. Nevertheless the hyperbolic model accounted for a fairly large proportion of the variation in yield at two of the three sites. Moreover, the rather small proportion of the variation in yield accounted for by each of the models at the third site, Winterton, was not so much because of a poor fit as because the range in mean yield was rather small at this site, being between 6944 and 8109 kg ha^{-1}. Thus, it can be seen (Figure 1) that differences between measured and predicted yields were not exceptionally large at Winterton. Figure 1 shows further that the differences between measured yields and those predicted by the hyperbolic equations were usually not large, although the quadratic equations gave generally better predictions at all three localities.

TABLE 3 Quadratic and hyperbolic equations relating grain yield (kg ha^{-1}) to nutrient application, together with the multiple correlation coefficient, r, and the R^2 value, i.e. fraction of the variation in yield accounted by the equation.

Site	Quadratic	Hyperbolic
Normandien	$Y = 2395{,}6 + 25{,}8N + 40{,}7P - 0{,}094N^2 - 0{,}285P^2 + 0{,}139NP$ $r = 0{,}94 \quad R^2 = 88\%$	$Y = (0{,}000111 + 0{,}00306/N + 0{,}00139/P)^{-1}$ $r = 0{,}90 \quad R^2 = 80\%$
Winterton	$Y = 5849{,}2 + 3{,}77N + 42{,}3P - 0{,}0105N^2 - 0{,}313P^2 + 0{,}0248NP$ $r = 0{,}64 \quad R^2 = 40\%$	$Y = (0{,}000123 + 0{,}00011/N + 0{,}000535/P)^{-1}$ $r = 0{,}55 \quad R^2 = 30\%$
Dundee	$Y = 3573{,}8 + 30{,}7N + 26{,}5P - 0{,}0817N^2 - 0{,}122P^2 + 0{,}102NP$ $r = 0{,}89 \quad R^2 = 78\%$	$Y = (0{,}0000747 + 0{,}00306/N + 0{,}00101/P)^{-1}$ $r = 0{,}85 \quad R^2 = 72\%$

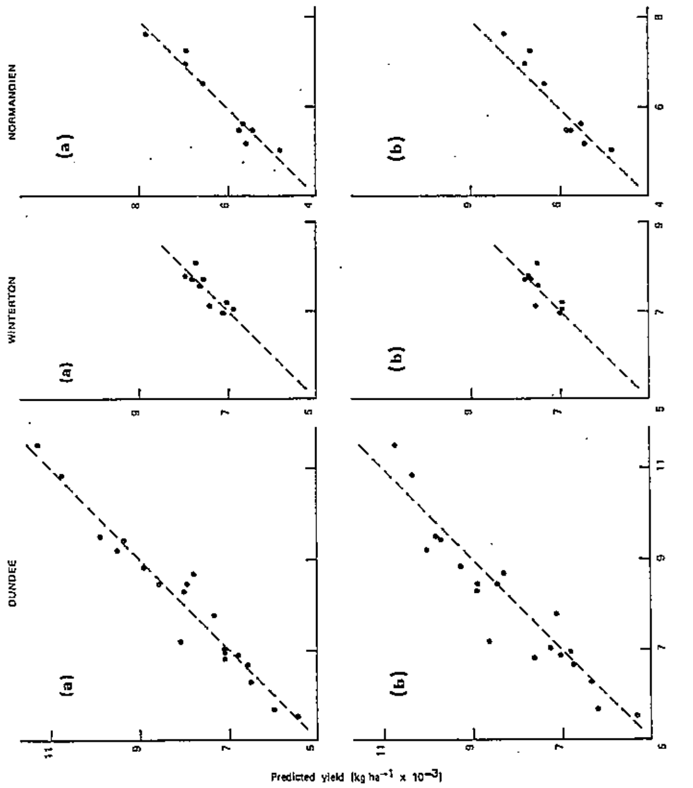

FIG 1 Yields predicted by (a) quadratic and (b) hyperbolic response surfaces plotted against measured yields in field experiments at three sites. Diagonal broken lines represent points of no difference between measured and predicted yields. Data from experiments reported by Farina et al *(1975).*

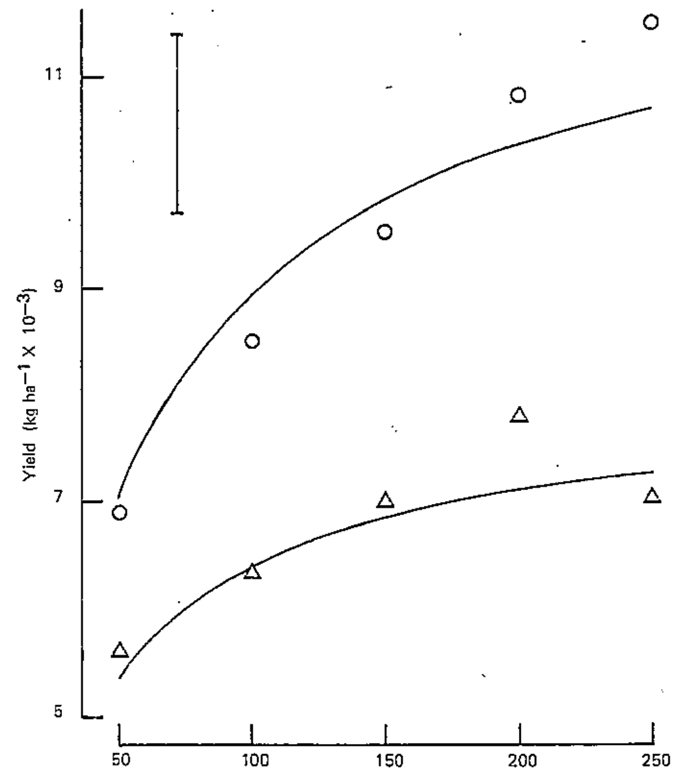

FIG 2 Response of yield to increasing amounts of nitrogen at two levels of phosphorus, Δ, 20 kg ha^{-1}, and O, 160 kg ha^{-1}, at Dundee, together with fitted curves calculated from the hyperbolic equation for this site in Table 3. The vertical bar indicates the least significant difference (P = 0,05) between individual yields.

Discussion

The hyperbolic model is capable of exhibiting the two features necessary for the usual economic assessment of fertilizer applications (cf Jones and Hocknell, 1962), namely curvature in the response of yield to increasing quantities of a nutrient, and interaction between the two independent variables, i e a 'steeper' response to one nutrient when a greater amount of the other is applied (see Figure 2). However, the hyperbolic relationship did not give as good fits to the measured yields as did the quadratic relationship. On the other hand the hyperbolic surfaces could have been derived with less effort than the quadratic surfaces. Not only are fewer plots required with the hyperbolic model, but because of the comparative simplicity of the treatments the management of an experiment should be less onerous. Accordingly it should be practicable to carry out a comparatively large number of experiments of this sort with the available resources in an agricultural region, and for practical decision-making purposes, the less accurate hyperbolic relationship determined on the site could be more significant than the more precise quadratic relationship determined at a distant site. At the same time the range of conditions included in such a series of experiments should clearly not be so wide that it is uncertain whether the two levels of a nutrient chosen are within the appropriate part of the yield-nutrient response curve, namely where diminishing returns are exhibited. So, such experiments should probably be regarded mainly as supplementing the conventional factorial fertilizer experiment on the experimental station, i e as a means of adjusting the quantities of nutrients applied to meet the individual circumstances of farmers within the general area served by the experimental station. Appropriate levels of a nutrient for the type of simple experiment being proposed might often simply be 0,5 X and 1,5 X where X is the level in current use on a particular farm.

Too little evidence has been considered to reach a clear conclusion on the usefulness of the proposed method. Further tests of the method against experimental data could give both poorer and better fits than those obtained here. It would be desirable to examine the use of the method with suitable data from a variety of conditions.

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Opsomming

'N METODE OM OPBRENGS-PLANTVOEDSEL REAKSIEVLAKKE UIT EENVOUDIGE BEMESTINGSPROEWE TE HERLEI.

Beide die kwadratiese, $Y = a+bX_1+cX_2+dX_1^2+eX_2^2+fX_1X_2$, *en die reghoekig-hiperboliese,* $Y = (a' + b'/X_1 + c'/X_2)^{-1}$, *regressievlakke kan die reaksie van opbrengs op toenemende hoeveelhede van twee plantvoedingstowwe,* X_1 *en* X_2, *bevredigend beskryf op voorwaarde dat plantvoedselpeile tot die gebied beperk word waar 'dalende meeropbrengs' voorkom. Die gebruik van die relasie,* $Y^{-1} = a' + b'X_1^{-1} + c'X_2^{-1}$, *wat 'n plat oppervlak is, maak dit moontlik om die hiperboliese regressie te herlei uit slegs vier behandelings, nl 'n 'lae' en 'n 'hoë' peil van* X_1 *gekombineer met 'n 'lae' en 'hoë' peil van* X_2. *Beide die kwadratiese en die plat vorm van die hiperboliese model is deur die metode van kleinste vierkante gepas op resultate van drie stikstof- en fosforproewe met mielies. Passing was in elke geval ietwat beter met die kwadratiese as met die hiperboliese regressies. Die eenvoud en die tipe proef wat vereis word om hiperboliese reaksievlakke te herlei behoort dit moontlik te maak om inligting vir die skatting van ekonomiese optimumhoeveelhede kunsmis van 'n redelik groot aantal plekke en seisoene te verkry.*

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